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Application of Mathematical Analogies Enables Shedding Useful Light on Some Critical Aerospace-Safety Problems: Perspective and Examples

E Suhir*

Bell Labs, Basic Research, Murray Hill, NJ (ret); Portland State University, Portland, OR, and University of California, Los Angeles, CA, United States of America

***Corresponding Author:** E Suhir, Bell Labs, Basic Research, Murray Hill, NJ (ret); Portland State University, Portland, OR, and University of California, Los Angeles, CA, United States of America, Tel.: 650-969-1530, E-mail:suhire@aol.com

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Abstract

The employment of appropriate mathematical analogies enables, in addition to other effort, shedding useful light on some critical aerospace-safety tasks and problems by providing additional and effective information for predicting the most likely outcome of a mission or an off-normal situation. Some well-known, "classical", analogies are indicated, and several recently introduced ones are addressed and briefly discussed. These particularly include:

1) analogies that have to do with some critical space-safety human-in-the-loop (HITL) type problems in electronics, reliability physics and ergonomics engineering, when the reliability of the instrumentation's, its hard- and software, and human performance contribute jointly to the outcome of a critical mission or a possible extraordinary situation; 2) analogies associated, first of all, with the human factor, and particularly with the current (temporary) state of human health or mind that might have an effect on his/hers failure-free performance and possibly result in a human error (HE); and 3) analogies between the probabilistic interpretation of the (originally deterministic) information-theory-based Fitts' law in the theory of human-computer interactions (HCI) and some critical outer space navigation phenomena, such as the likelihood of a spacecraft collision with an asteroid, or the re-occurrence of the famous Tunguska-meteorite type of an event. The general concepts are illustrated by numerical examples. It is concluded that while some kind of predictive modeling should always be conducted prior to and, when possible and appropriate, also during accelerated reliability testing, analytical ("mathematical"), preferably probabilistic, modeling, such as the one based on mathematical-analogies, should complement computer simulations. Computer simulations and the "old fashioned" analytical modeling are based on different assumptions and use different calculation techniques, and if the results obtained using these two major available modeling tools are in agreement, then there is a good reason to believe that the obtained data are sufficiently accurate and, hence, trustworthy. Future work should consider other suitable applications of the taken approach, as well as practical methodologies for establishing the ultimate risks for particular critical applications: the levels of such risks should take into account, of course, not only the probabilities of the anticipated critical failures, but the possible consequences of such failures as well.

Keywords: Boltzmann-Arrhenius-Zhurkov (BAZ) equation; Double-exponential-probability-distribution-function (DEPD-F); Failure-oriented-accelerated-testing (FOAT); Figure-of-merit (FoM); Finite-element-analyses (FEA); Highly-accelerated-life-testing (HALT); Human-capacity-factor (HCF); Human-computer interactions (HCI); Human error (HE); Human-in-the-loop (HITL); Human non-failure (HnF); Human-system-interaction/integration (HSI); Mean-time-to-error (MTTE); Mean-time-to-failure (MTTF); Mental (cognitive) workload (MWL); Probabilistic-design-for-reliability (PDfR) concept; Probabilistic predictive modeling (PPM); State-of-health (SoH)

Introduction

Predictive modeling, both computer simulation based and analytical ("mathematical"), should always complement each other in every critical research undertaking to make sure that the output results are in agreement, so that the obtained information is sufficiently accurate and trustworthy. As Abraham Maslow (1908-1970), a famous American psychologist, has indicated (Maslow's "law of instrument"), "if the only tool you have is a hammer, you tend to see every problem as a nail". The emphasis of this paper is on a special type of analytical modeling – mathematical analogies. The application of this type of modeling, while is not a must and, in addition, is not always possible, is nonetheless and often highly desirable: as it is said in the title of our paper, it enables shedding useful additional light on some critical problems, tasks, missions or situations. Although there are no obvious gaps in the existing aerospace safety models where the use of mathematical analogies is inevitable, such a use might be valuable, when the "analogous" phenomenon or a feature lends itself to simple, inexpensive and trustworthy experimental or theoretical investigations, analyses and implementations.

In the analyses suggested in this write-up two new, powerful, flexible and physically meaningful constitutive equations are considered as suitable examples: 1) multi-parametric Boltzmann-Arrhenius-Zhurkov (BAZ) equation, when the system of importance experiences, in addition to other stressors, thermally induced loading, and 2) the double-exponential-probability-distribution-function (DEPDF), related to BAZ, when there are no thermal stresses among the anticipated acting stimuli. The DEPDF is employed, particularly, in the human-in-the-loop (HITL) example. This is a practically important situation, when the reliability of the instrumentation, both its hard-and software, and the performance of the human(s), if any, contribute jointly to the outcome of a critical safety problem. Finally, we suggested a probabilistic interpretation of the originally deterministic Fitts' law, suggested by Dr. Fitts about forty years ago in the theory of human-computer-interactions (HCI), and use this interpretation to develop a useful analogy-based model for two critical aerospace safety related problems: 1) the likelihood of a spacecraft collision with an asteroid and 2) the probability of occurrence of another Tunguska-meteorite type of event.

Analytical ("Mathematical") Modeling and Mathematical Analogies

Analytical ("Mathematical") Modeling

Analytical ("mathematical") modeling has been employed in all the addressed analogies. Of course, as has been indicated, predictive modeling of some type should always be considered and conducted prior to and, whenever possible and appropriate, also during the actual accelerated tests of any kind. It is also desirable that analytical ("mathematical") modeling, preferably, because of inevitable intervening uncertainties, of the probabilistic-design-for-reliability (PDfR) [1, 2] type, should complement computer simulations.

Analytical predictive modeling occupies a special place in the reliability-physics modeling effort [3, 4]: such modeling enables obtaining relationships that clearly indicate "what affects what", but, more importantly, can often explain the physics of phenomena and particularly various paradoxical situations [5-7] better than computer simulations or even experimentation, can. As Heinrich Hertz (1857-1894), the famous German physicist has put it, "mathematical formulas have their own life, they are smarter than we, even smarter than their authors, and often provide more than what has been expected from them." Computer-aided finite-element-analyses (FEA), e.g., initially implemented in the mid-1950s in the areas of engineering where structures of complicated geometry were employed (aerospace, maritime, some civil engineering structures), has become shortly, owing to the progress in computer science, the major modeling tool in electronics and photonics engineering and in reliability physics as well: powerful and flexible FEA computer programs enable obtaining, within a reasonable time, a solution to almost any stress-strain related problem. Analytical solutions in various reliability physics problems are, however, still important: simple, easy-to-use and physically meaningful analytical relationships provide clear and compact information of the role of various factors affecting the phenomenon or the behavior and performance of a material or a device of interest.

A crucial requirement for an effective analytical model is simplicity, clear physical meaning, and, as Einstein has put it, "external justification and internal perfection". A good analytical model should be based on physically meaningful considerations and produce simple and easy-to-use relationships, clearly indicating the role of the major factors affecting the technology, the phenomenon, the object or the structure of interest. One authority in applied physics remarked, perhaps only partly in jest, that the degree of understanding a physical phenomenon is inversely proportional to the number of variables used for its description and that an equation longer than two inches is most likely wrong. Hooke's law $\sigma = E\varepsilon$ *(1678)* in the strength-of-materials, Newton's second law $F = ma$ (1687) in mechanics ("natural philosophy") and Einstein's $E = mc^2$ *(1905)* relationship in his relativity theory are, probably, the best illustrations to this statement. Empirical relationships, such as, e.g., the Coffin-Manson's ones in the reliability of solder-joint interconnections in electronics and photonics engineering, are certainly useful, but the structure of these relationships and particularly their non-integer exponents clearly indicate on the lack of understanding the underlying physics of their failure.

Classical Mathematical Analogies

As Gottfried Wilhelm Leibnitz (1646-1716), the famous German mathematician, had put it, "there are things in this world, far more important than the most splendid discoveries – it is the methods by which they were made". Mathematical analogies are one of such methods. This method belongs, of course, to the class of analytical ("mathematical") modeling tools. Analogies compare "two otherwise unlike things based on resemblance of a particular aspect" (Merriam-Webster) and have helped in the past, as is known, making sense of things in many areas of applied science and engineering. This form of reasoning assumes that "if two things are known to be alike in some respect(s), then they are probably alike in other respects" (Free Dictionary) as well. Such reasoning might be and, in effect, has been helpful when trying to understand an unknown phenomenon, a new technology or a new design or a particular hidden feature of importance by using its formal mathematical resemblance, "analogy", with another, reasonably well known and/or easier to investigate, theoretically or experimentally, "analogous" phenomenon or a feature. Such an opportunity is particularly valuable, when the "analogous" phenomenon or a feature lends itself to simple, inexpensive and trustworthy experimental investigations and analyses.

Here are some meaningful and useful "classical "examples of mathematical analogies successfully employed in the past in various fields of applied science and engineering: the structure of an atom and the solar system ("Bohr's atom") [8]; the flow of electrical charge (current) through a wire and the flow of fluid through a pipe; small elastic measurable displacements of a thin membrane subjected to lateral pressure and sought stresses in a twisted thin-wall bar ("Prandtl's membrane analogy") [9]; the efficiency of falling water and temperature difference in steam engines ("Carnot engine") [10]; the motions of a mathematical pendulum and large deflections of flexible elastic rods (such as, e.g., optical silica fibers) [11]; some two-dimensional problems in hydrodynamics and in the mathematical theory of elasticity (Muskhelishvili's theory) [12]; Boltzmann's equation in classical thermodynamics of ideal gases [13] and Arrhenius' equation in physical chemistry [14], Arrhenius equation and Zhurkov's equation in experimental fracture mechanics of solids [15, 16]; Fitts' law in the human-computer-interaction (HCI) [17] and Shannon's information theory [18, 19].

Perspective

Probabilistic Design for Reliability Concept

The probabilistic-design-for-reliability (PDfR) concept [1, 2, 22] that was applied to the human-in-the-loop (HITL) aerospace (see, e.g., [21, 22]) and other vehicular missions and situations, is based on the failure-oriented-accelerated-testing (FOAT) technique [23, 24]. FOAT at the design stage is supposed to be conducted when a new technology, or a new design, or a new application of an existing technology or a design is considered and when no acceptable highly-accelerated-life-testing (HALT) [25, 26] procedures exist yet, nor suitable "best practices" have been established and agreed upon, and when there is a need and an intent to evaluate the useful lifetime of a product and the corresponding probability of its field failure. This probability is, in effect, never zero, but, using the PDfR concept and the design-stage FOAT, could be made low enough to be adequate for the given product and application. The recently suggested multi-parametric Boltzmann-Arrhenius-Zhurkov (BAZ) equation [27-29] can be used to predict the probability of failure from the design-stage FOAT data. It is noteworthy that this type of FOAT should be considered in addition to the manufacturing-stage-FOAT, known as "burn-in-testing (BIT)" (see, e.g., [3, 4, 30]) and routinely conducted for any electronic, photonic or MEMS product, and also in addition to the development-stage-FOATs, such as, e.g., shear-off testing or temperature cycling.

The development-stage FOAT is conducted to make sure that the considered technological and design approach and materials selection are acceptable, and the BIT type FOAT is conducted to get rid of "freaks", low reliability products, prior to shipping the healthy ones, i.e., those that survived BITs, to the customer(s). It has been shown [29] that the multi-parametric BAZ model can be applied not only to the electronic packages and systems, but also to electronic devices, where the reliability of the p-n junction is critical. The multi-parametric BAZ equation (see next section for details) was initially suggested for the prediction of lifetime of IC packages and devices [27, 28] and then applied, as a suitable analogy, in electronic manufacturing [30], space biology [31], ergonomics [32-35], medical-and-clinical [36-39] problems with an objective to establish the required/adequate level of the human capacity factor (HCF) [40-42] in various ergonomics-engineering human-in-the-loop undertakings. In this analogy, the activation energy (the "strength") in the BAZ equation plays the role of the HCF (the human's "bearing capacity") in the ergonomics-engineering formulation, and the thermal energy, defined in the BAZ equation as the product of the Boltzmann' constant and the absolute temperature, reflects the role and the level of the mental (cognitive) workload (MWL) [33, 34, 39]. Challenges that an aircraft pilot faces in an extraordinary situation are more or less analogous to those that a surgeon copes with during an operation [38, 39]. Analogies associated with the role of the human factor and the state of his/hers health were addressed in [40].

Simple experiments based on the probabilistic interpretation of the deterministic Fitts' law in the theory of human-computer interactions (HCI) could be conducted, considering their analogy with some critical outer space phenomena. In this write-up we consider as suitable examples the likelihood of a spacecraft collision with an asteroid [43] and the probability of the Tunguska-meteorite type of an event [44]. As Don Yeomans, Manager, Near-Earth Object Office, NASA's Jet Propulsion Laboratory, Pasadena, CA, USA, put it, "If you want to start a conversation with anyone in the asteroid business, all you have to say is "Tunguska".It is the only entry of a large meteoroid we have in the modern era with first-hand accounts." Analogies that have to do with various human-in-the-loop (HITL) situations, when instrumentation's reliability and humans' performance contribute jointly to the outcome of a critical aerospace mission or an off-normal situation, were considered in detail in [42-44], as well as analogies between the probabilistic interpretation of the originally deterministic [17] information-theory-based Fitts' law in the theory of human-computer interactions [43-45] and some relatively recent critical outer space navigation phenomena, such as the likelihood of a spacecraft collision with an asteroid [43], or the re-occurrence of the famous Tunguska-meteorite type of an event [44, 48-50] that took place about a century ago.

Multi-Parametric Boltzmann-Arrhenius-Zhurkov (BAZ) Equation

When FOAT is conducted, a physically meaningful constitutive equation, such as the multi-parametric Boltzmann-Arrhenius-Zhurkov (BAZ) model [22-24]

$$P = \exp\left[-\gamma_C C t \exp\left(-\frac{1}{k_B T}\left(U_0 - \sum_{i=1}^{n} \gamma_i \sigma_i\right)\right)\right] \quad (1)$$

for the probability of non-failure, could be employed to interpret and to quantify the test data. In this equation σiis the i-th stressor, γ_i is its sensitivity factor, *C* is the continuously monitored and measured (during the FOAT procedure) response (such as, e.g., leakage current or electrical resistance or any other suitable and measurable feedback) of the system and γ_c is the sensitivity factor for this response. The response provides information about the degree of degradation (current damage level) during the FOAT, and the remaining "distance" (time, damage) from its a-priori decided-and-agreed-upon level $C= C_*$ viewed as an adequate indication of failure. The model can be obtained by combining Boltzmann's distribution [13] in classical thermodynamics, Arrhenius equation in physical chemistry [14] and Zhurkov's extension [15, 16] of the Arrhenius equation in experimental fracture mechanics. As has been shown [27], the equation (1) can be also obtained as a steady-state solution to the Fokker-Planck equation in the theory of Markovian processes. The appropriate stressors in (1) could be any stimuli that shorten the useful lifetime of a device, package, module or a system of interest.

Non-Thermal Look at the BAZ Equation

But let us take a "not-necessarily-thermal" look at the BAZ [27-29] double-exponential equation. In such a situation the probability-of-non-failure reflects, first of all, the role of the ratio of the element's (material's, system's, human) bearing capacity (analogous to the "activation energy") to the external loading (analogous to the "thermal energy"). When applied to an individual human, the function (1) reflects the role of the ratio of his/hers human-capacity-factor (HCF) (which is analogous to the "activation energy" that characterizes the "bearing capacity" of a system) to the mental/cognitive workload (MWL) (analogous to "thermal energy" that characterizes the level of the loading on the system). In the recently suggested "probabilistic Fitts' law" [43, 44] the probability

$$P = \exp\left[-\lambda t \exp\left(-\frac{W}{2D}\right)\right] \quad (2)$$

of non-failure, i.e., the probability of hitting the target - the black rectangular on the computer screen, increases with the increase in the width *W* of this rectangular (which is analogous to the "activation energy", the "bearing capacity" of the target) and decreases with an increase in the distance *D* from the user to the computer screen (which is analogous to "thermal energy", the "loading"). As to a particular individual's "human quality", the human capacity factor (HCF), the great Russian writer Leo Tolstoy, "God's elder brother" [51], made a rather broad statement about the human quality regardless of a particular situation or application: "A man is like a fraction whose numerator is what he is and the denominator is what he thinks of himself; the larger the denominator - the smaller the fraction". Thus, the above double-exponential-probability-distribution-function (DEPDF) (see, e.g., [22, 23]) is a rather general, flexible, broad, physically meaningful and useful probabilistic quantitative description that is applicable to many physical, "human-in-the-loop" (HITL), ergonomics, medical and clinical systems.

Examples

Predicted Probability of an Aerospace Mission Success and Safety

Analogies that have to do with some critical space-safety HITL type problems in reliability-physics and ergonomics- engineering, when the reliability of the instrumentation's and human performance contribute jointly to the outcome of a critical aerospace mission or a possible extraordinary situation, have been addressed in [20-22, 32-35, 41, 42, 45-47]. While improvements in safety in the air and in space can be achieved through better ergonomics, better work environment, and other efforts of the traditional avionic psychology that directly affect human behaviours and performance, there is also a significant potential for further improvements in aerospace safety through better understanding the roles that various uncertainties play in the planner's and operator's worlds of work, when never-perfect human, never failure-free navigation equipment and instrumentation, never hundred-percent-predictable response of the object of control (air- or spacecraft), and uncertain-and-often-harsh environments contribute jointly to the likelihood of a mishap. By employing quantifiable and measurable ways of assessing the role and significance of such uncertainties and treating a HITL as a part, often the most crucial part, of a complex man-instrumentation-equipment-craft-environment system, one could improve dramatically the state-of-the-art in assuring aerospace safety. This can be done by predicting, quantifying and, if necessary, even specifying an adequate (of, course, low enough) probability of a possible accident. Nothing and nobody is perfect, and the difference between a highly reliable object, product or a mission and an insufficiently reliable one is "merely" in the level of their never-zero probability of failure. Application of the probabilistic predictive modeling and probabilistic design for reliability concepts provide a natural and an effective means for reduction vehicular casualties. It is noteworthy that while the traditional statistical human-factor-oriented approaches are based on experimentations followed by statistical analyses, the PPM and PDfR concepts are based on, and starts with, physically meaningful and flexible predictive modelling followed by highly focused and highly cost effective experimentations geared to the chosen governing model(s). The PPM and PDfR concepts enable quantifying, on the probabilistic basis, the outcome of a particular HITL related mission. If the predicted outcome, in terms of the most likely probability of the operational failure, does not seem to be acceptable for the given mission, then an appropriate sensitivity analysis based on the developed and available calculation procedures can be effectively conducted to improve the situation. With the appropriate modifications and generalizations, such a cost-effective and insightful approach is applicable to numerous, not even necessarily in the aerospace and vehicular domain, HITL related missions and situations, when a human encounters an uncertain environment or a hazardous off-normal situation. The approach is applicable also when there is an incentive to quantify human's qualifications and performance, and/or when there is a need to assess and possibly improve his/hers role in a particular mission or a situation.

While the model (1) can be and, as a matter of fact, has been used to quantify the likelihood of the human non-failure [22, 32, 40-42, 45-47], the reliability of the equipment (instrumentation), including the performance of both the hardware and the software, can be characterized also by Weibull distribution, which is, as is known, widely used in reliability engineering. Let, e.g., a particular aerospace mission of interest consists of n segments (so that $i=1,2,\ldots n$) characterized by different probabilities, q_i, of occurrence of a particular harsh environment or some other extraordinary conditions during the fulfillment of the mission/-flight at the *i-th* segment. The segments are characterized also by different durations, T_i, and possibly by different failure rates, λ_i^e of the equipment and instrumentation. These rates may or may not depend on the environmental conditions, but could be affected by aging/degradation and other time-dependent causes. In the simplified example below we assume that the combined input of the hardware and the software, as far as the performance of the equipment and instrumentation is concerned, is evaluated beforehand and is adequately reflected by the appropriate available failure rate λ_i^e values. These could be either determined from the vendor specifications or, preferably, should be obtained on the basis of the specially designed and conducted failure oriented accelerated testing (FOAT) [23, 24] and the subsequent modeling. FOAT should be preferably geared to a particular predictive model, such as, e.g., BAZ model [27-30] applied previously to electronic or photonic devices, packages, modules and systems subjected to thermal loading (in addition to the non-thermal ones) or, also analytical, exponential Weibull distribution. Let the probability of the equipment non-failure at the moment t_i of time during the fulfillment of the mission on the *i-th* segment, assuming that Weibull distribution is applicable, be

$$P_i^e = \exp\left[-(\lambda_i^e t_i)^{\beta_i^e}\right] \quad (3)$$

Here $0 \leq t_i \leq T_i$ is an arbitrary moment of time within the *i-th* segment, and β_i^e is the shape parameter in the distribution. One could assume that the time-dependent probability of human non-failure can be also represented in the form of Wei bull distribution

$$P_i^h(t_i) = P_i^h(0)\exp\left[-\left(\lambda_i^h t_i\right)^{\beta_i^h}\right] \quad (4)$$

Here λ_i^h is the failure rate, β_i^h is the shape parameter and $P_i^h\ (0)$ is the probability of the human non-failureat the initial moment of time ti =0 of the given segment. When ti →∞, the probability of non-failure (say, because of the humanfatigue or other causes) tends to zero. The probability $P_i^h\ (0)$ can be assumed particularly in the form of the distribution(1). The probability of the mission failure at the i-th segment can be found, in an approximate analysis (in a more rigorous anal-ysis conditional probabilities should be considered) as

$$Q_i(t_i) = 1 - P_i^e(t_i)P_i^h(t_i) \quad (5)$$

and the overall probability of the mission failure can be determined as

$$Q = \sum_{i=1}^{n} q_i Q_i(t_i) = 1 - \sum_{i=1}^{n} q_i P_i^e(t_i)P_i^h(t_i) \quad (6)$$

This formula can be used also for specifying the failure rates and the HCF in such a way that the overall probability of failure would be adequate for the given mission. The assessments based on the formula (6) can be used to choose, if possible, an alter-native route or time, so that the set of the probabilities q_i of encounter the environmental conditions of the given severity brings the overall probability of the mission failure to an acceptable sufficiently low low level.

Let, for instance, the duration of a particular vehicular mission be 24 hours, and the vehicle spends equal times at each of the 6 segments (so that $t_i=4$ hours at the end of each segment), the failure rates of the equipment and the human performance are in-dependent of the environmental conditions and are $\lambda = 8x10^{-4}$ 1/hour, the shape parameter in the Weibull distribution in both cases is $\beta=2$ (Rayleigh distribution is applicable), the HCF ratio $\frac{F^2}{F_0^2}$ is $\frac{F^2}{F_0^2} = 8$ (so that $\frac{F}{F_0} = 2.828$),

The probability of human non-failure at ordinary conditions is $P_0 = 0.9900$, and the MWL G_i^2/G_0^2 ratios are 1, 2, 3, 4, 5, and occur with the probabilities q_i =0.9530, 0.0399, 0.0050, 0.0010, 0.0006 and 0.0005. These data indicate that about 95% of the mis-sion time takes place in ordinary conditions. The calculated $\bar{P}_i = \frac{P_i^h(t_i)}{P_i^h(0)}$ ratios for the six segments are 1.0000; 0.9991; 0.9982; 0.9978; 0.9964 and 0.9955. The computed probabilities P_i^h of the human non-failures are 0.9900; 0.9891; 0.9882; 0.9878; 0.9864 and 0.9855. The products $P_i^e P_i^h$ of the equipment and the human non-failures are 0.9900; 0.9891; 0.9882; 0.9878; 0.9864 and 0.9855, and the products $q_i P_i^e P_i^h$ are 0.9435; 0.0395; 0.0049; 0.0010; 0.0006; and 0.0005. With these data the predicted probability of the mission's non-failure is $P = \sum_{i=1}^{n} q_i P_i^e(t_i)P_i^h(t_i) = 0.9900$, and the predicted probability of failure is therefore $Q = 0.01 = 1\%$.

The Current (Temporary) State of Human's Health and/or Mind and Its Role in Avoiding Human Error

The current (temporary) state of human health or mind during the fulfillment of an aerospace mission or when encountering an extraordinary situation could affect his/hers failure-free performance resulting in what is known as human error. Let us ad-dress, as a suitable example of an analogy based medical application of the probabilistic predictive modeling, probabilistic hu-man-system-interaction/integration (HSI) in aerospace engineering, or, specifically, navigator's (aircraft pilot's or astronaut's) performance vs. his/hers human-capacity-factor (HCF). The emphasis is on his/hers state-of-health (SoH). The following dou-ble-exponential-probability distribution function (DEPDF) for the probability of human-non-failure, when performing a mis-sion of importance, or when encountering an off-normal, but not an impossible, situation, could be assumed in the form [40]:

$$P^h(F,G,S_*) = P_0 \exp\left[\left(1-\gamma_S S_* t - \frac{G^2}{G_0^2}\right)\exp\left(1-\gamma_T T_* - \frac{F^2}{F_0^2}\right)\right]. \quad (7)$$

This function enables evaluating the impact of three major factors, the mental workload (MWL) G the human capacity factor (HCF) F, and the time t (possibly affecting the navigator's performance, such as, e.g., the likelihood of making a mistake, and sometimes even affecting his/hers health), on the probability (7) of the human-non-failure. Here P_0 is the initial probability *(t=0)* and at a normal (sufficiently low) level of the MWL $(G=G_0)$, S_* is the threshold (acceptable level) of the (supposedly continuously monitored/measured, cumulative, effective, indicative, and possibly even multi-parametric) health ("medical") characteristic, such as, say, body temperature, arterial blood pressure, oxyhaemo-metric determination of the level of saturation of blood hemoglobin with oxygen, electrocardiogram measurements, pulse frequency and fullness, frequency of respiration, measurement of skin resistance that reflects skin covering with sweat, etc. etc. (since the time t and the threshold S_* enter the above governing expression as a product $S_* t$ each of these parameters has a similar cumulative impact on the sought probability); γ_s is the sensitivity factor for the symptom S_*; $G \geq G_0$ is the actual (elevated, off-normal, extraordinary, possibly even time-dependent) MWL; G_0 is the MWL at ordinary (normal) operation conditions; T_* is the mean time to error/failure (MTTF); γ_T is the sensitivity factor for this time; $F \geq F_0$ is the actual (could be off-normal) HCF exhibited or required in a particular condition/situation of importance; F_0 is the most likely (normal, specified, ordinary) HCF. There is a certain overlap, of course, between the levels of the HCF F and the MTTF T_* values: both have to do with the human quality and performance. The difference is, however, that T_* is a short-term characteristic of the navigator's performance that might be affected, first of all, by his/hers personality and vulnerability to various influences, while the HCF is a long-term characteristic, such as his/hers age, education, experience, ability to think and act independently and under pressure, and, if necessary, as a team player, etc. etc.

The MTTF T_* might be determined for the given individual by using a highly focused failure-oriented-accelerated-testing (FOAT) on a flight simulator [34], whatever the appropriate definition of failure in such testing might be, while the HCF F, which should also be quantified, cannot obviously be evaluated experimentally and should be quantified using a F specially designed methodology. It is noteworthy also that while the P_0 value is defined as the probability of the human-non-failure at a very low MWL level G it could be determined and evaluated also as the probability of the human-non-failure for a hypothetical situation, when the HCF F is extraordinarily high, i.e., for a navigator who is exceptionally highly qualified (like, say, Captain "Sully" in the famous "miracle-on-the-Hudson" event [42]), while the MWL G is still finite, and so is the operation time t. The suggested governing DEPDF function has a nice symmetric form. Indeed, it reflects the roles of the "objective", "external", MWL plus the state- of-health (SoH) impact

$$E = \left(1 - \gamma_s S_* t - \frac{G^2}{G_0^2}\right) \quad (8)$$

as well as of the "subjective"," internal", human capacity factor (HCF) plus the likelihood of human error HCF+HE impacts

$$I = \left(1 - \gamma_T T_* - \frac{F^2}{F_0^2}\right). \ (9)$$

Here is the rationale below the structures of these expressions. The level of the MWL could be affected by the human's SoH: the navigator might experience a higher MWL, which is not only different for different individuals, but might be quite different for the same individual, depending on his/hers current, short-term, SoH, while his/hers HCF, although could also be influenced by the state of his/hers SoH, affects the probability of the human non-failure (HnF) indirectly. In our approach the impact of the human's state-of-health (SoH) could be measured/quantified by the navigator's mean-time-to-error (MTTE) T_*, since the human error (HE) is, in effect, a failure, interruption, in his/hers otherwise error-free performance process, is it not? When the human's qualification is high, the likelihood of an error is most likely low, regardless of how harsh the external conditions are. Thus, in our model the "external" factor E=MWL+SoH (mental workload plus state-of-health) is a more or less short-term characteristic of the human performance, while the "internal" factor I=HCF+HE (human capacity factor plus propensity to make an error) is a more permanent, a long-term characteristic of the navigator's HCF. It is also noteworthy that the human's mind (reflected by his/hers MWL) and his/her body's SoH are closely linked, that such link is different for different individuals, and that is at present far from being more or less clearly understood and well defined. The suggested formalism is, of course, just a possible and a highly tentative way to account for such a link. Difficulties may arise in some particular occasions when the MWL and the SoH factors overlap. It is anticipated therefore that the MWL impact in the suggested formalism considers, to an extent possible, various more or less most important influences other than the direct SoH related ones.

Human capacity factor (HCF), unlike mental/cognitive workload (MWL), is a relatively new notion in ergonomics engineering (see, e.g., [33]). HCF plays with respect to the MWL approximately the same role as strength/capacity plays with respect to stress/demand in structural analysis and in some economics problems. HCF includes, but might not be limited to, the following major qualities that would enable a professional human to successfully cope with an elevated off-normal MWL: age; fitness; health; personality type; psychological suitability for a particular task; professional experience and qualifications; education, both special and general; relevant capabilities and skills; level, quality and timeliness of training; performance sustainability (consistency, predictability); independent thinking and independent acting, when necessary; ability to concentrate; awareness and ability to anticipate; ability to withstand fatigue; self-control and ability to act in cold blood in hazardous and even life threatening situations; mature (realistic) thinking; ability to operate effectively under pressure, and particularly under time pressure; leadership ability; ability to operate effectively, when necessary, in a tireless fashion, for a long period of time (tolerance to stress); ability to act effectively under time pressure and make well substantiated decisions in a short period of time and in an uncertain environmental conditions; team-player attitude, when necessary; swiftness in reaction, when necessary; adequate trust (in humans, technologies, equipment); ability to maintain the optimal level of physiological arousal. These and other qualities are certainly of different importance in different human-in-the-loop (HITL) situations. It is clear also that different individuals possess these qualities in different degrees. Long-term HCF could be time-dependent. To come up with suitable figures-of-merit (FoM) for the HCF, one could rank, similarly to the MWL estimates, the above and perhaps other qualities on the scale from, say, one to ten, and calculate the average FoM for each individual and particular task (see, e.g., [22, 32, 33-42] . Clearly, MWL and HCF measurements should use the same units, which could be particularly non-dimensional. Special psychological tests might be necessary to develop and conduct to establish the level of these qualities for the individuals of significance.

In connection with the taken approach it is noteworthy also that not every model needs prior or even posterior experimental validation. In the author's view, the structure of our governing models does not. Just the opposite: this model should be used as the basis of the FOAT to establish the MWL, HCF, and various human errors (HE) through the corresponding observed and recorded MTTF and his/hers SoH at normal operation conditions and for a navigator with regular skills and of ordinary human capacity. These experiments could be conducted, e.g., on flight simulators [34] and using various specially developed testing methodologies. Being a probabilistic, not a statistical model, the approach should be used to obtain, interpret and to accumulate relevant statistical information. Starting with collecting statistics first seems to be a time consuming and highly expensive path often leading to nowhere. The possible FOAT procedure to establish the suitable double-exponential-probability-distribution-function of the reliability physics type, including detailed numerical examples, could be found in publications [34, 45].

Analogies Between the Probabilistic Fitts' Law and Two Critical Space-Safety Problems

As has been indicated above, the original Fitts' law in the theory of human-computer interaction is based on and is analogous to the Shannon's information theory. This law was deterministic, i.e., non-random. The recently suggested probabilistic extension of Fitts' law [43, 44] can be applied as a suitable analogy to shed useful light on two critical outer space navigation problems: the likelihood of a spacecraft collision with an asteroid and the possible re-occurrence of the famous Tunguska meteorite event.

Problem #1: The Likelihood of a Spacecraft Collision with an Asteroid

The original Fitts' law [17]

$$ID = \log_2\left(\frac{2D}{W}\right) \quad (10)$$

suggested by Dr. Fitts in 1954 was based on Shannon's information theory [18, 19] published six years earlier. Considering the state of the art of the human-computer interaction (HCI) technology-and-practices at that time, Fitts suggested an "index of difficulty (ID)" as a figure of merit required to rapidly move to the target area on the computer screen. The index ID in (10) is a function of the ratio $\frac{2D}{W}$ of the doubled distance D (considering the feedback) to the center of the target to the target's width W. The "law" was later on applied by human psychologists and ergonomics engineers to many non-HCI tasks and problems as well. The author of this write-up suggested [43, 44] a probabilistic interpretation of the deterministic Fitt' law [17] considering the state of the HCI practice at that time: the governing ratio $\frac{2D}{W}$ in this interpretation a random variable. While the index ID is just a useful figure of merit, the probability of failure of a human interacting with a computer or another intelligent device has a clear physical meaning. As Laplace, French mathematician and founder of the applied probability, has put it, "probability theory is nothing but common sense reduced to calculation”. The DEPDF (2) considers the probability

$$Q = 1 - \exp\left[-\lambda t \exp\left(-\frac{W}{2D}\right)\right] \quad (11)$$

of the human failure to complete, when using the original Fitts' law (10), the required action during the given time t. Here λ is the sensitivity parameter for this time and is, in effect, the failure rate of the random HCI process. In the model (10) the width W of the target is, in a way, analogous to the HCF and the distance *D* - to the MWL (see, e.g., [33]). In the Fitts' law (10) the width W is analogous to the signal (favorable factor) and the distance *D* - to the noise (unfavorable factor). The probability of non-failure *P = 1 - Q* is expressed by the formula (2). Note that the entropy *H(P) = - PlnP* of this probability distribution reaches its maximum $H_{max} = e^{-1}$ at $P=e^{-1}$. The time derivative $\frac{dP}{dt} = -\frac{H(P)}{t}$ of the probability (2) reflexes the physical rationale underlying this distribution: this derivative is always negative (the probability of non-failure decreases with time) and increases with an increase in the level of the entropy H(P). The time

$$t = \tau = \frac{1}{\lambda} \exp\left(\frac{W}{2D}\right) \quad (12)$$

when the entropy *H(P)* reaches its maximum value, is, in accordance with the Arrhenius theory [14], the mean-time-to-failure (MTTF). As evident from (11), this time increases with an increase in the $\frac{W}{2D}$ ratio.

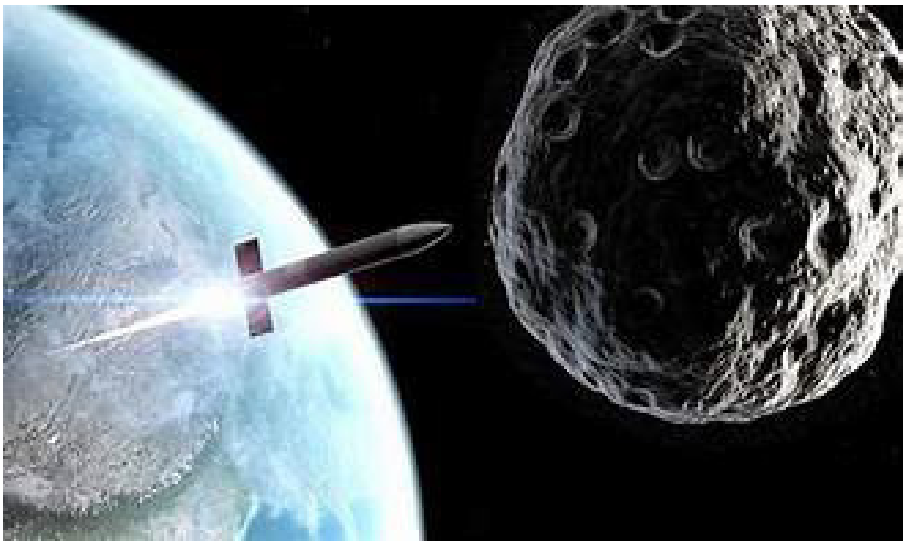

Figure 1: Spacecraft heading towards an asteroid .

The equation (2) makes physical sense. Indeed, if the width *W* of the target is large, the probability of non-failure, i.e., the probability of hitting it, is significant. When this width is next-to-zero, the probability of non-failure could still be significant, if the product λt is small, i.e. if either the time to reach the target, or the failure rate of the process, or both, is small, otherwise the probability (2) is low, no matter how large the ratio $\frac{W}{2D}$ might be. These intuitively obvious behaviors are quantified, on the probabilistic basis, by the equations (2) and (10).

These equations have just one unknown - the time sensitivity factor λ. This factor could be found from an appropriate FOAT [23], and here is how this could be done. Let's carry out FOAT with the same individual (to exclude the role of the human factor) and for the same width W of the target, but for two different distances D and two appreciably different times t, so that one could establish with sufficient accuracy that the human under test failed hitting the target times out of M_1 attempts for the established distance D_1 during the time t_1, and failed hitting the same target m_2 times out of M_2 attempts for the distance D_2 during the time t_2. This means that his/hers probability of failure was Q_1 for the time t_1 and the distance D_1 and was Q_2 for the time t_2 and the distance D_2. Because the width of the target was kept the same in both sets of tests, the condition

$$D_1 \ln\left(\frac{n_1}{\lambda}\right) = D_2\left(\frac{n_2}{\lambda}\right), \quad n_{1,2} = \frac{-\ln[(1-Q_{1,2})]}{t_{1,2}} \qquad (13)$$

should be fulfilled. This condition, viewed as an equation for the failure rate λ, yields:

$$\exp\left(\frac{D_1 \ln n_1 - D_2 \ln n_2}{D_1 - D_2}\right) = \exp\left[\frac{1}{D_1 - D_2}\ln\left(\frac{{n_1}^{D_1}}{{n_2}^{D_2}}\right)\right]. \quad (14)$$

After the factor λ, is determined, the probability Q and the MTTF can be found using the formulas (10), and (11). Consider a numerical example. Let, for instance, the first set of FOAT indicated that the individual under test failed $m_1=10$ times out of $M_1=12$ attempts, when the distance to the target was $D_1=1m$, and the duration of each test was 2s. This means that his/hers probability of failure was $Q_1 = \frac{10}{12} = 0.8333$. When the distance to the target was changed to $D_2 = 0.75m$ and the test time - to 1s, the human under test failed 7 times out of $M_2 = 10$ attempts, so that the probability of failure was $Q_2 = \frac{7}{10} = 0.7000$. Then we have:

$$n_1 = \frac{-\ln[(1-Q_1)]}{t_1} = \frac{-\ln 0.1667}{2} = 0.8959, n_2 = \frac{-\ln[(1-Q_2)]}{t_1} = \frac{-\ln 0.3}{1} = 1.2040,$$

and the failure rate of the process is

$$\lambda = \exp\left[\frac{1}{D_1 - D_2}\ln\left(\frac{{n_1}^{D_1}}{{n_2}^{D_2}}\right)\right] = \exp\left[\frac{1}{0.25}\ln\left(\frac{0.8959^{1.0}}{1.2040^{0.75}}\right)\right] = 0.3621s^{-1}.$$

Table 1: Calculated probabilities Q of human failure for different width-to-distance ratios and times

$\frac{W}{2D}$	0.05	0.1	0.2	0.3	0.4	0.5	1.0	5.0	10.0
t, s									
1	0.2914	0.2794	0.2566	0.2353	0.2155	0.1972	0.1247	0.2437E-2	1.6439E-5
2	0.4979	0.4807	0.4473	0.4152	0.3846	0.3555	0.2339	0.4868E-2	3.2878E-5
3	0.6442	0.6258	0.5891	0.5528	0.5172	0.4826	0.3294	0.7293E-2	4.9317E-5
4	0.7479	0.7303	0.6945	0.6580	0.6213	0.5846	0.4131	0.9713E-2	0.6576E-4
5	0.8213	0.8057	0.7729	0.7385	0.7029	0.6665	0.4863	1.2126E-3	0.8219E-4
6	0.8734	0.8600	0.8312	0.8000	0.7669	0.7323	0.5503	1.4533E-4	0.9863E-4

Then the equation (10) results in the following probability of failure:

$Q = 1 - \exp\left[-0.3621 t \exp\left(-\frac{W}{2D}\right)\right]$. The calculated data in Table 1 indicate that this probability slowly decreases with an increase in the W/2D ratio and increases with time. This intuitively obvious situation is quantified by the calculated data.

Excluding the $\frac{2D}{W}$ ratio from the Fitts' law (10) written as

$$ID = \log_2\left(\frac{2D}{W}\right) = \frac{1}{\ln 2}\ln\left(\frac{2D}{W}\right) = 1.4427\ln\left(\frac{2D}{W}\right) \quad (15)$$

and the distribution (2), the following triple-exponential expression for the probability of non-failure can be obtained:

$$P = \exp\left[-\lambda t \exp\left(-\exp\left(-(ID)\ln 2\right)\right)\right] = \exp\left[-\lambda t \exp\left[-\exp\left(-0.6931(ID)\right]\right] \quad (16)$$

The calculated data in Table 2 show, particularly, how fast the probability $P = 1 - Q$ of non-failure of the effort in question decreases with an increase in the ID index and the product λt of the failure rate λ of the effort and time t.

The Fitts' law (10) treated as a probabilistic process could be applied to assess the likelihood of collision of a spacecraft with an asteroid (Figure 1), considering the asteroid's size ("width"), W, its initial (detected) distance, D, from the spacecraft and the estimated time t to the possible collision.

Table 2: Probability of non-failure as a function of the index of difficulty ID and the product λt of the failure rate of the Fitts' relationship, treated as a random process, and time

$ID = \log_2\left(\frac{2D}{W}\right)$ / λt	0	1	2	3	4	10	∞
0	1.0						
1	0.6922	0.5452	0.4590	0.4138	0.3909	0.3682	0.3679
2	0.4791	0.2973	0.2106	0.1712	0.1528	0.1356	0.1353
3	0.3317	0.1621	0.9668E-1	0.7083E-1	0.5931E-1	0.4993E-1	0.4978E-1
4	0.2296	0.8838E-1	0.4437E-1	0.2931E-1	0.2334E-1	0.1839E-1	0.1832E-1
10	0.2525E-1	0.2322E-2	0.4148E-3	0.1470E-3	0.8322E-4	0.45845E-4	0.4540E-6
∞	0						

It is important to emphasize that in this application, unlike in the original, "classical", deterministic, Fitts' law suggested in the HCI field seventy years ago, a significant probability Q of "missing the target"-the asteroid- is highly desirable and so is the time of reaching it: if this time is long enough, then there might be enough time to either redirect the spacecraft or, if possible, even to destroy the asteroid. Because of that, in this application, unlike in the original Fitts' law, the probability of missing the target is "a success", and the probability of hitting it - is "a failure".

Let us revisit the calculated data in Tables 1 (indicating the probability of success) and 2, having in mind the likelihood of collision of a spacecraft with an asteroid. The probabilities of success in Table 1 are greater for lower $W/2D$ ratios (a small size asteroid is located far enough from the spacecraft) and for longer times t to reach the "target". This intuitively obvious situation is quantified by the calculated data. For small size asteroids located at not very small distances from the aircraft, when the ratio $W/2D$ is small compared to 1, the formula (11) yields $Q=1-exp(-\lambda t)$. This is a well known exponential law of reliability.

The Fitts' index ID, a useful "figure of merit", is substituted in our probabilistic model with the DEPDF for the probability of failure to complete the required action during the given time. The model adequately reflects the reliability physics underlying the random process of hitting a target and should complement Fitts' law in various computer, ergonomics, medical and other applications. The likelihood of a spacecraft collision with an asteroid is considered as a useful application of the probabilistic Fitts' law in astronautics. The analogy with the HITL situation could be used in this case as a way to establish the failure rate of the random process of avoiding the spacecraft's collision with an asteroid - the undesirable "target".

Problem #2: The Likelihood that a Big Asteroid Becomes a Meteorite and Hits Earth

Actually, this is an attempt to assess, using the probabilistic interpretation of the Fitts' law, the likelihood of the re- occurrence of the famous Tunguska meteorite event [44] (Figure 2). As Don Yeomans, Manager, NASA's Jet Propulsion Laboratory, Pasadena, CA, USA, has put it, "If you want to start a conversation with anyone in the asteroid business, all you have to say is "Tunguska". It is the only entry of a large meteoroid we have in the modern era with first-hand accounts."

Figure 2: Asteroid has become a meteorite and is about to hit Earth

Two extreme cases of the situation in question are considered here: 1) a big asteroid that travels within the solar system is still far away from Earth (the probabilistic Fitts' law reduces in such a case to the well-known exponential law of reliability) and 2) the asteroid has become a meteorite and is approaching Earth (the complete double-exponential probabilistic Fitts' law is employed in such a case). The addressed analogy can be used, in addition to other efforts, to design, conduct and interpret an inexpensive, but, of course, also highly tentative HCI-theory-based imitation, viewed, however, as analogous to the vital space safety related problem, the Tunguska event [48-54], the largest outer space impact event on Earth in the recorded history. No wonder that analyses of this event occupy a special place among numerous studies addressing collision between objects in space. An explosion of the magnitude of this event, if had occurred over a populated region or over an ocean, would be able to destroy a large metropolitan area or trigger a devastating tsunami. The objective of our analysis is, however, not to try to shed more light on what had most likely happened above the Russian taiga a century ago, but, considering that the Tunguska meteorite was, actually, a big asteroid, a giant stony "meteoroid", to suggest, using an analogy of the event with the probabilistic interpretation of Fitts' law [44], an independent, inexpensive and, of course, a highly tentative way for assessing the likelihood and the corresponding time frame of a Tunguska type event, i.e., the probability and the corresponding time that a big asteroid travelling within the solar system becomes eventually a meteorite and might hit Earth.

This is what actually happened 115 years ago, on June 30, 1908. A bright sky object entered the Earth atmosphere from the east--southeast direction. Fortunately (God is merciful!), that happened in almost unpopulated area of Eastern Siberia, Russia, in a remote coniferous forest ("taiga") region (Figure 3) between the rivers Lena and Podkamennaya ("Stony") Tunguska, a tributary of the Yenisei river. The object was seen to fly (Figure 4), likely with a speed of approximately 27 km/s (60,000 mph), about 320 km (200 miles) and then exploded ("disintegrated") in the air at an altitude of 5 to 10 km (3 to 6 miles). It has been estimated later on that the magnitude of the explosion was about 15 megaton. Megaton is, as is known, a unit of explosive power used today mostly for nuclear weapons and is equivalent to one million tons of TNT (trinitrotoluol) of a regular explosive. This magnitude is roughly a thousand times that of the atomic bomb dropped in 1945 on Hiroshima. The thunderclaps of the explosion were heard over thousands of miles. The produced shock waves were detected even in England. Newspapers reported at that time that this may have been a volcanic explosion or a mining accident or – a far-fetched idea – an asteroid or a comet that arrived from the outer space. As a result of the blast, the forest within 50km (30 miles) or so felled (Figure 5). The explosion flattened an estimated 80 million trees over an area of 2,150 km^2 (830 sq mi or 500,000 acres) of forest, the size of a metropolitan city today: the city of Los Angeles, e.g., covers a total area of 502.7 sq mi, the NYC area is 319.0 sq mi, and the area of the city of Philadelphia is only 135 sq mi. The area was devastated by the subsequent fire. For many years after the blast the taiga in the region became a cemetery for numerous perished animals and woods, as well as some people.

Figure 3: Tunguska event location

Figure 4: Tunguska meteorite in Earth atmosphere

Figure 5: Eighty million trees were flattened by the shock wave

There are many hypotheses and a number of theories of what has actually happened. The general consensus today is that the Tunguska meteorite was a large space rock, about 400m (120 feet) across. New research from NASA [52-54] indicates that the impacts of the relatively mid-size rocks of the Tunguska meteorite type might be, however, much less frequent than previously thought: such impacts might happen on the order of millennia – not centuries.

As to the Tunguska event problem, as has been indicated, two major extreme situations of the probabilistic Fitts' law are considered in this write-up: 1) a situation when a big asteroid that travels within the solar system, but is still far away from Earth (in such a case the probabilistic Fitts' law reduces to the well-known exponential law of reliability) and 2) a situation when the time to the possible impact and the distance between the Earth and the asteroid are relatively short, so that the asteroid has a potential to become a meteorite and hit Earth. Examine these two situations in some detail, assuming that because of a short distance between the asteroid and Earth, the asteroid's elliptic trajectory does not have to be accounted for.

A Big Asteroid is Traveling within the Solar System, But Is Still Far Away from Earth

The probabilistic version of the Fitts' law (2) could be applied, e.g., for the assessment of the likelihood of the Tunguska event considering the asteroid's size ("width"), W, its initial (detected) distance, D from Earth and the estimated time t to the possible impact. The calculated failure rates

$$\lambda = \lambda_A = \frac{-\ln P}{t}, s^{-1} \quad (17)$$

Table 3: HCI failure rates λ_A, S^{-1}, corresponding to the probability P_A that the remote asteroid eventually becomes a meteorite and hits Earth after the given time

P_A	10^{-1}	10^{-2}	10^{-3}	10^{-5}	10^{-10}	10^{-15}	10^{-20}
$-InP_A$	2.3026	4.6052	6.9078	11.5129	23.0259	34.5388	46.0517
t,years	HCI failure rates reflecting the probability P_A that the remote asteroid eventually becomes a meteorite and hits Earth: for the same probability the rates are higher for shorter times						
100	4.381E-8	8.762E-8	1.314E-7	2.190E-7	4.381E-7	6.571E-7	8.762E-7
500	4.381E-9	1.752E-8	2.628E-8	4.381E-8	8.762E-8	1.314E-7	1.752E-7
1000	2.190E-9	8.762E-9	1.314E-8	2.190E-8	4.381E-8	6.571E-8	8.762E-8
5000	8.762E-10	1.752E-9	2.628E-9	4.381E-9	8.762E-9	1.314E-8	1.752E-8

of such a random process are shown in Table 3. This table indicates, e.g., that, as long as the exponential law of reliability is applicable, and the Table 3 data for the expected failure rates λ were obtained using an appropriate and trustworthy HCI experimental technique, the detected asteroid that is still far away from Earth could become a meteorite and enter the Earth atmosphere in 100 years with the probability 10^{-5}, if the failure rate obtained using HCI FOAT is 2.190E-7, and in 1000 years with the probability 10^{-5}, if the failure rate obtained using HCI FOAT is 6.571E-8, etc. The Table 3 data can be used particularly to organize and conduct the appropriate HCI FOAT. As long as the formulas (15) and (16) is applicable, one does not have, of course, to organize and conduct numerous HCI accelerated tests, but, perhaps, just a couple of them to confirm the validity of the table data and the applicability of the formula (17) for the failure rate.

A Big Asteroid is Detected at a Short Distance from Earth and is Likely to Become a Meteorite

The equation (2) results in the following formula for the failure rate λM of a meteorite (note that in our approach higher failure rates mean higher probability of "hitting the target", i.e. higher probability of an impact: this probability is certainly higher in the case of a meteorite than in the case of an asteroid traveling at a significant distance from Earth):

Table 4: Correction $\exp\left(\frac{W}{2D}\right)$ for the case, when an asteroid becomes a meteorite

$\frac{W}{2D}$	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	5.0
$\exp\left(\frac{W}{2D}\right)$	1.6481	2.7183	4.4817	7.3891	12.1825	20.0855	33.1155	54.5982	148.4132

Where λA is the failure rate, when the space body is still an asteroid. This failure rate is expressed by the formula (18). As evident from the formula (11), if an asteroid, located at the distance 2D from Earth is approaching Earth,

then the Table 3 data should be multiplied by $\exp\left(\frac{W}{2D}\right)$ to obtain the failure rates of the random process of hitting

the Earth by such an asteroid that has now become a meteorite. The function $exp\frac{W}{2D}$ is tabulated in Table 4.

Let, e.g., the HCI FOAT conducted for a situation when the computer screen is relatively far away from the users has indicated that the failure rate of the HCI process is, say, λ_A =4.381E-9 S^{-1}. As follows from the Table 3 data, this means that the likelihood that this Tunguska type asteroid becomes a meteorite and hits Earth in 5000 years is only 10^{-5}. If, however, this asteroid has become already a meteorite and is detected at the distance from our planet equal, say, to its diameter of $W = 6{,}378km$, then the corresponding failure rate of the analogous HCI process should be multiplied by $\exp\left(\frac{W}{2D}\right) = e = 2.7183,$ and is λ_M=1.191E-8 S^{-1}. Then, as follows from Table 6 data, the probability that this meteorite hits Earth in, say, "only" 500 years is as "high" as only 10%. Obviously, after each new observation, new orbital parameters and new impact predictions are inferred, and therefore also new values for Lambda should be obtained.

A model that interprets Fitts' law from the probabilistic standpoint is suggested and applied to the famous Tunguska meteorite event. The addressed analogy can be used to design, conduct and interpret an inexpensive HCI-theory-based imitation of a vital space problem. The following related problems will be addressed: possible collision with small debris, satellite-to-satellite collision (the problem of "overcrowding"), or a "collision cascade" that could possibly develop creating, as Dr. John W. Evans, NASA, has indicated in his comment to the author, an untenable situation. The author would like to acknowledge also, with thanks, very useful comments made by the reviewer. Another possible application, which is also considered as future work and is, certainly and literally, of "down-to-Earth" type, is the probability of vehicular obstruction with a steadfast obstacle (Figure 6).

Figure 6: Vehicle heading towards a steadfast obstacle

Conclusion

The article presents a more or less detailed exploration of how mathematical analogies can provide, in addition to other modeling and experimentation efforts, valuable insights into some critical aerospace safety problems. In the author's judgment, this is the key takeaway from using suitable mathematical analogies. While some kind of predictive modeling should always be conducted prior to and, when possible and appropriate, also during accelerated reliability testing, analytical ("mathematical"), preferably probabilistic, modeling, such as the one based on mathematical-analogies, should complement computer simulations. As has been indicated, computer simulations and analytical ("mathematical") modeling are based on different assumptions and use different calculation techniques, and if the results obtained using these two major modeling tools are in agreement, then there is a good reason to believe that the obtained data are sufficiently accurate, adequate and, hence, trustworthy. Future work should consider other possible applications of the addressed suitable-analogies-based approach and a methodology for establishing the ultimate risks, having in mind that the levels of such risks should consider not only the predicted probabilities of the anticipated critical failures, but the consequences of such failures as well.

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